## A Model of Farm Settlement

The agent, a potential settler, is assumed to maximize lifetime utility subject to an income constraint. If the agent does not settle a farm, he receives a constant income, $y^*$, over his lifetime, 0 to T. Alternatively, the agent may choose to settle a farm at year $t_0$, in which case he receives $y^*$ until that time and an income after $t_0$ that depends on the amount of land cleared. The relationship is described by:

$$\text{(1a)} \qquad y(t) = \begin{cases} y^* & t \leq t_0 \\ 1 - e^{-p(t-t0)} & t > t_0. \end{cases}$$

The pattern of income after the farm is settled, $t > t_0$, is based on the view that the amount of agricultural output depends on the area of improved land. At $t = t_0$, no land has been cleared, so farm income is zero. But as land is cleared at the rate p, income begins to rise. At first, all the settler's time is devoted to clearing, but as the area of improved land increases more time is spent farming. Agricultural output thus increases at a decreasing rate, approaching asymptotically the normalized value of 1, which represents the income generated by a fully cleared farm.

A crucial feature of the model is the assumption that a settler is unable to borrow against future farm income. Thus, to maintain consumption during the early years of farm making, a settler must save before settlement and then draw down those savings once the farm is broken. Clearing thus involves not just a change in the pattern of income the person receives but also a change in the consumption stream. A settler must sacrifice consumption in the early years in return for higher rates of consumption later. This idea is formalized by assuming that the agent receives utility each period:

$$\text{(2a)} \qquad u(t) = u[c(t)]e^{-\rho t} \qquad u'>0,\ u''<0,$$

where c is consumption and $\rho$ is the pure rate of time preference. The agent is constrained not to borrow, but he may save at discount rate r. Assuming r and $\rho$ are the same, it follows that consumption will be constant throughout his life if he chooses not to settle.[1] If he does settle, consumption will be constant while he is saving, that is, prior to $t_0$, and consumption will continue at the same rate while these savings are being drawn down. Only after enough of the farm has been cleared will consumption begin to rise.

The agent faces two related decisions: when to settle and how much to consume before and during the early years of settlement. Finally, assuming settlement time and consumption are chosen optimally, the agent compares lifetime utility depending on whether or not he settles. The formal lifetime optimization problem follows:

$$\text{(3a)} \qquad \max_{t_0, c(t)} U = \int_0^T u[c(t)]e^{-\rho t}dt,$$

subject to,

$$\text{(4a)} \qquad \int_0^n y(t)e^{-rt}dt \geq \int_0^n c(t)e^{-rt}dt,\ 0 \leq n \leq T$$

where income, $y(t)$, is described by equation (1a).[2] The income constraint, equation (4a), ensures non-negative aggregate savings at each age, which is consistent with the assumption that settlers cannot borrow against future income. Assuming the potential settler starts a farm at time $t_0$, he will save during the years up to $t_0$ and then run down the accumulated savings during the early years of settlement. The settler's problem can be decomposed into two periods, the years up to some period $t_1$, when the borrowing constraint is not binding, and the time after $t_1$, when it is binding. Assuming the pure rate of time preference and the discount rate are the same, optimization implies a constant rate of consumption up to $t_1$. After that, consumption will equal the rising farm income. These standard life-cycle results allow the optimization problem to be rewritten:

[1] Assuming r and $\rho$ are the same simplifies the presentation without affecting the main implications of the model.
[2] One could moddify the lifetime utility function to allow for a bequest motive.

$$\text{(5a)} \qquad \max_{t_0, c^*} U = \int_0^{t_1} u(c^*)e^{-\rho t}dt + \int_{t_1}^{T} u[y(t)]e^{-\rho t}dt + \lambda \int_0^{t_1} [y(t)-c^*]e^{-rt}\,dt,$$

where $c^*$ is optimal consumption during the period of saving and dissaving, and $t_1$ is the point at which the farm generates income equal to $c^*$. The first-order condition with respect to $c^*$ is:

$$\text{(6a)} \qquad u'(c^*) = \lambda,$$

which says simply that the lagrange multiplier of the problem is the marginal utility of consumption at time o. The first-order condition with respect to settlement time, $t_0$, is more revealing. It requires:

$$\text{(7a)} \qquad -\rho e^{\rho t_0}\int_{t_1}^{T} u'[y(t)]e^{-\rho t}dt + \lambda\{y^* e^{-rt_0} - \rho e^{\rho t_0}\int_{t_0}^{t_1} e^{-(\rho+r)t}dt\} = 0.$$

Equation (7a) illustrates the trade-off associated with the decision about when to settle. If settlement time is postponed one period, the settler gains income $y^*$ during the period prior to $t_1$, offset partly by lower income owing to the delay in starting the farm. The net gain, multiplied by the marginal utility of consumption, $\lambda$, which is given by the last two components of the equation, is compared to the loss of utility after $t_1$. Again, because the farm is started one period later, the farmer receives less income and, hence, enjoys less utility from $t_1$ to $T$. This loss is described by the first term of equation (7a). At the optimal settlement time, the two effects are equal. Finally, assuming $c^*$ and $t_0$ are chosen optimally, the potential settler compares lifetime utility in the states where he does and doesn't start a farm:

$$\int_0^{T} u(y^*)e^{-\rho t}dt \quad vs \quad \int_0^{t_1} u(c^*)e^{-\rho t}dt + \int_{t_1}^{T} u[y(t)]e^{-\rho t}dt.$$

The model has been simulated using parameter values that are roughly consistent with the early economy of Upper Canada. To begin, the agent's preferences are characterized by a Stone-Geary utility function. This specification allows for the introduction of a subsistence level of consumption. In addition, the function is assumed isoelastic. Thus:

$$\text{(8a)} \qquad u(c) = \begin{cases} (c-s)^{1-\delta}/1-\delta & \delta \neq 1,\ \delta > 0 \\ \ln(c-s), & \delta = 1, \end{cases}$$

where s can be interpreted as subsistence consumption, and $\delta$ is the elasticity of marginal utility with respect to consumption. In the simulations, $\delta$ is put at 1, and s is assumed 0.2.[3] Both the pure rate of time preference and the discount rate are assumed 0.03. The discount rate is lower than contemporary mortgage and bond rates, although, given that this is intended as a net lending rate adjusted for any risk premium, it does not seem unrealistic. The pure rate of time preference is perhaps slightly above values assumed in contemporary work, which may be appropriate given the long-term decline in mortality rates. Finally, the time horizon $T$ is put at thirty-five years.

A barrier to settlement was the initial cost of the land, farm equipment, and draft animals. For the purpose of illustration, the cost of this capital equals the annual income of a fully cleared farm. Although this is a modest capital cost, the simulation indicates that no agent, regardless of alternative income, would find it optimal to settle. Consider an agent with income $y^*$ of 0.30. He would save for nearly thirteen years before settling, and the income returned in the remaining time available is not enough compensation for the period of lower consumption.

To illustrate the possible impact of revenue from lumber on the settlement decision, a further simulation has been run, which includes an income of 0.1 each year following initial settlement.[4] The additional income makes it optimal for those with incomes between 0.32 and 0.52 to settle. The simulation supports the hypothesis suggested by the theoretical model as well as the historical literature that the poorest will not settle because settling is not the optimal choice for anyone with an alternative income below 0.32. Note that, with the exception of those near the top of the income range, the incomes of settlers will eventually more than double. Finally, an initial capital requirement causes settlement to be delayed: the initial period of saving is roughly ten years, even longer for those with low income. Thus, settlers would not be expected to start farms until they are at least age twenty-five to thirty unless they have other sources of capital.

[3] We take a typical completed farm to have 50 improved acres generating $250 in agricultural output annually. Based on the normalization in our model, an income of $250 is assigned a value of 1. Thus, subsistence consumption is by implication $50. Fifty dollars is roughly equal to per capita income in the United States at this time, a period when average household size was more than 5 persons. Contemporary estimates of $\delta$, which can also be interpreted as the measure of risk aversion, are about 2. It might be argued that the people who chose to settle were probably less risk averse than the average. The level of subsistence consumption assumed is possibly on the low side, given that the consumption unit is the farm.

[4] Assuming a completed farm generates $250, forestry income would amount to an additional $25 per year.